Collective decisions with interdependent valuations*

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Abstract Many collective decision problems have the common feature that individuals' desired outcomes are correlated but not identical. This paper studies collective decisions with private information about these desired policies. Each agent holds private information which mainly concerns his own bliss point, but this private information also affects all other agents' bliss points. We concentrate on two specific mechanisms, the median and the mean mechanism. We establish existence of two symmetric Bayesian Nash equilibria of the corresponding game and compare the performance of the mechanisms for different degrees of interdependencies. Applications of our framework include the provision of public goods and the design of decision processes in teams, firms, and international organizations.

Keywords: collective decisions, asymmetric information, interdependent valuations, public goods

JEL N.: D78, D82, H41

1 Introduction

1.1 Decisions with public values

Many collective decision problems have in common that there is some agreement between the individuals who are supposed to take decisions as well as some disagreement. Our paper models such situations in an environment of asymmetric information by including interdependencies between individual preferences. This means that the individually preferred decision of a group member does not only depend on his own private information but also on the other members' private information. The questions are how the decision mechanism should be designed and how existing mechanisms perform when individual preferences are correlated.

We consider a specific class of collective decision problems where each of these has the following properties: In order to take a common decision, all agents obtain private information (a signal) about their most desired policy. However, no individual is perfectly informed about what would be the privately optimal policy. This imperfection is due to spillover effects between the desired policies. The information of all individuals could be used to calculate the private bliss points whereby each individuals' private information yields more information about its own bliss point than any other individual's private information. Decision problems are characterized by one single parameter which measures the extent to which private information affects all individuals.

In this setting we analyze a specific class of mechanisms. Participation is not voluntary, therefore we can ignore any individual rationality constraints. Moreover, our mechanisms do not condition monetary transfers on the agents' announcements, in fact all monetary transfers are ruled out a-priori. Instead, the mechanisms map individual announcements of the private information into the collective decision.

We concentrate on two mechanisms, i.e. the median and the mean mechanism. The median mechanism implements the median announcement, whereas the mean mechanism implements the average of all announcements. The main difference between these two mechanisms is how they deal with the announcements of private information. Under the median mechanism changes in extreme positions are disregarded, since the median alone determines the final decision. On the contrary, the nature of the mean mechanism is to take all available information into account. Therefore, under the mean mechanism extreme positions influence the decision.

The main result of this paper is the identification of two symmetric Bayesian Nash equilibria of the respective games. The performance of the mechanisms depends upon the extent to which spillover effects affect the economy. With weak interdependencies, the median mechanism dominates the mean mechanism, whereas with strong interdependencies it is optimal to use the average as decision mechanism.

1.2 Some applications

One can think of a number of different applications of our framework: Any setting in which the individually preferred decision does not only depend on the agents' own information but as well on the signals of the others fits well into this framework.

One example is the decision process in a common central bank like the European Central Bank (ECB). Here, national central banks may care about a policy that accommodates macroeconomic shocks in their own country while taking a collective decision about common monetary policy. However, due to demand spillover effects, shocks in one country may affect the desired policy in other participating countries. Most of the information used by national central bankers within the central bank council is common knowledge. Our model relates to a situation where at least part of the information of council members is their private information. Hard data is exchanged in the European System of Central Banks before decisions are taken in the Governing Council. This data concerns recent economic indicators, new figures on growth, employment etc. Nevertheless, there is some soft information available to central bank governors which cannot be communicated. This information arises partly from the national central bank governors' experience in dealing with national data. Actually, this is one justification why national central bankers should play a role in the decision process. Private information may also concern expectations about the way in which other major macroeconomic players such as governments or trade unions will react to certain central bank decisions. If national central bankers are experts in this sense then there may be scope for manipulation of information. If interdependencies are strong, the other central bankers' information is important for the nationally desired policy.

Our setting can also be applied to other international decisions such as decisions about environmental policy. Basically, the nation states are interested in achieving less pollution in their own country. On the other hand, they have to take a common decision about certain environmental standards. In addition, it may be the case that national governments possess private information about the national amount of emissions, the costs to reduce emissions or the economic consequences of a reduction. However, the environmental situation in one country is co-determined by the emissions in the neighboring countries. The nearer the location of countries, the more important becomes private information obtained in any single country. Take as a recent example the negotiations about a common European standard of reduction in CO_2 emissions in preparation for the Kyoto Protocol.

Another important application of our framework is the collective decision on the provision of a public good. Imagine a situation where information about the desirability of a certain public good comes in two forms: (i) a measure of individual desirability of this particular good and (ii) a quality component that affects the desirability of all individuals in the same way. Suppose that each agent receives a combined signal which is related to his private component and to the quality of the good.[1] In this case preferences about the provision of the public good are correlated but not identical.[2]

Collective decision problems having the features described above can be found also in many areas besides politics. Consider the following example taken from industrial organization: a decision about the future orientation of a firm has to be made. This decision has to be taken by the different heads of department. First of all, these heads are interested in the performance of their own department. Beside this, they possess specific knowledge about the conditions, needs or prospects of it. However, their opinion about the future development of the firm is influenced by the conditions obtaining in other departments as well.

1.3 Relation to literature

The mechanism design literature offers solutions to related problems, but none to our specific setting. If there are no spillovers and side-payments are allowed it is always possible to obtain (Bayesian) incentive-compatibility using an expected externality mechanism.[3] If informational and allocational externalities are considered but monetary transfers are allowed, the problem of efficient design has been analyzed in

[1]Another, possibly more complex and more appropriate, way to model quality components in public good decisions would be to consider two signals, one on individual preferences and the second on quality.

[2]In our model the desired policy is linear in all agents' signals. This particular specification arises in the public goods example when the desired quantity of the public good is a convex combination of the signal and the quality component and when the quality component is linearly related to all agents' private signals.

[3]See Mas-Colell, Whinston, Green [1995], Arrow [1979] or D'Aspremont and Gérard-Varet [1979].

auction environments.[4] Instead, we study the case where spillovers are present and monetary transfers are excluded a priori. Moreover, we do not consider or look for optimal mechanisms but concentrate on the performance of two specific mechanisms.

Concerning the analysis of collective decisions, the setting of our paper builds an intermediate case between two frameworks commonly used in the literature: On one hand, political outcomes under individual utility maximization are analyzed, i.e. the case of zero spillovers (see Vaubel and Willet [1991] and the references therein). On the other hand, the literature deals with efficient aggregation of perfectly co-ordinated interests, i.e. 100% spillovers (see Piketty [1999] and the references therein). Our research focuses on the intermediate case: what kind of political outcomes under different information aggregation mechanisms are to be expected if individual interests are correlated to a certain extent? Thus, we do not analyze political outcomes for a fixed degree of spillovers, but instead vary the extent to which individual preferences influence each other.

Related to our work is a recent paper by Casella [2000]. In a similar informational environment but with private values she proposes a voting scheme for deliberations taken by committees that meet regularly. At each meeting, committee members are allowed to store their vote for future use. Although the scheme cannot achieve the first-best with more than two voters, making votes storable typically leads to ex-ante welfare gains. Her paper differs from ours in that we do not consider developments over time. Instead, we study a one-shot game excluding also any reputation effects.

Our model differs from a common value setting where individuals receive imperfect signals about a desirable course of action in the following respect. An example of such a setting is the Condorcet jury problem where all members of a jury receive a signal about the true state of the world. All individuals would agree about the correct

[4]See for example Fieseler, Kittsteiner and Moldovanu [2000] or Jehiel and Moldovanu [1998].

decision once they are perfectly informed. In the common value setting individual information is correlated with the true state of the world. In our model, all signals are related to the desired decision in a deterministic manner. Hence, in our set-up individuals would still disagree about the collective decision when perfectly informed. Moreover, perfect information would obtain if all private signals were known.

The existing literature on decision making in a central banking context focuses either on monetary stabilization policies comparing alternative types of appointees (i.e. having different mandates) in the ECB council (e.g. Von Hagen and Süppel [1994]), on the implications of different policy objectives of the common central bank (e.g. Gros and Hefeker [2000] and Grüner [1999]) or on equilibrium incentive contracts in a multi-principal agency framework (e.g. Dixit and Jensen [2000]). Instead, we ask how the decision mechanism of the ECB should be designed if national central bankers possess private information and their preferences are partly correlated. Our result translates to this context in the following way: the larger the common component between member states, for example the more national macroeconomic shocks affect all members of the union, the more weight should be given to extreme positions in the ECB council.

The remainder of this paper is organized as follows: Section 2 introduces the model presenting the first-best solution and the two mechanisms we aim to study. The respective equilibria under those mechanisms are calculated in Section 3 where results about the truthful revelation properties of the equilibria are given as well. The next section compares the performance of the mean and the median mechanism for different degrees of interdependencies and develops a concrete example. The robustness of our results with respect to the specification of individual utility is analyzed in Section 5 and Section 6 concludes. All omitted proofs can be found in the Appendix.

2 The Model

We consider an economy which is populated by n individuals where n is an odd number. A decision $x \in \mathbb{R}$ has to be taken. Each individual receives private information θ_i. The parameters θ_i are distributed independently over the same support $\Theta \subseteq \mathbb{R}$. Let $f(\theta_i)$ denote the respective density. We assume that the expected value of all signals equals zero, i.e. $E[\theta_i] = 0 \; \forall \; i$. The vector of private information is θ. Individual preferences over outcomes are characterized by the following von-Neumann-Morgenstern utility function

$$u_i(x, \theta) = -(x - \theta_i^*)^2 \tag{1}$$

where θ_i^* describes the individually preferred decision. Individual utility is maximized at $x = \theta_i^*$ (i.e. when the individually preferred decision is actually implemented). The highest attainable utility is zero. The larger the difference between the implemented policy x and the individually preferred decision θ_i^*, the smaller is individual utility.

The individually preferred policy θ_i^* is a convex combination of $i's$ type and the average of all others' types, i.e.

$$\theta_i^* = (1 - \alpha)\, \theta_i + \frac{\alpha}{n-1} \sum_{j \neq i} \theta_j. \tag{2}$$

The parameter $\alpha \in [0, \bar{\alpha}]$ measures the extent to which interdependencies align the preferences of all individuals. For the upper bound value $\bar{\alpha} := \frac{n-1}{n}$ all individuals share a common utility function with a maximum at $x = \theta_{Mean}$ where $\theta_{Mean} = \sum_{i=1}^{n} \theta_i \,/\, n$. For the lower bound $\alpha = 0$ each individual has his signal θ_i as a private bliss point. For example, α might measure the importance of a common quality component of a public good, the degree of spillover effects between the members of the European Union, closeness of geographical location or the degree to which firm departments are interlinked.

2.1 Welfare

First, we describe the decision that maximizes welfare when there are no informational asymmetries. Welfare is defined as the sum of individual utilities, i.e. we take a utilitarian perspective. The sum of all utilities is maximized if x^* maximizes

$$\sum_{i=1}^{n} u_i(x, \theta) = \sum_{i=1}^{n} -(x - \theta_i^*)^2. \tag{3}$$

Optimality requires

$$-\sum_{i=1}^{n} 2(x^* - \theta_i^*) = 0. \tag{4}$$

Thus, it follows immediately that the decision x^* is independent of the degree of interdependency α and is determined by the average of all private signals.

Lemma 1 *The sum of all utilities is maximized at* $x^* = \theta_{Mean}$.

PROOF OF LEMMA 1:
Reorganizing (4) and substituting for θ_i^* yields the result. Q.E.D.

The intuition for this result is that by summing up the utilities of all individuals any spillover effects are automatically taken into account. Lemma 1 implies that the mean mechanism would yield the first-best if truth-telling could be implemented for all degrees of interdependency. However, if informational asymmetries are present, this first-best solution remains no longer attainable.

2.2 The mechanisms

We consider the following two direct mechanisms excluding any monetary transfers and ignoring participation constraints: All individuals are asked for an announcement $\hat{\theta}_i \in \mathbb{R}$ of their private signal. The vector of announcements is $\hat{\theta}$. Depending on these announcements the collective decision x is taken.

The first mechanism we study is the median mechanism. Let

$$x_{Median} = x_{Median}(\hat{\theta}) := \hat{\theta}_{Median} \tag{5}$$

where $\hat{\theta}_{Median}$ is the median of all announcements.[5] The median mechanism implements this announcement and thus replicates majority decisions for the case of zero spillovers. Another feature of this mechanism is that changes in extreme positions are disregarded, since the final decision solely depends on the median announcement.

The second mechanism we analyze is the mean mechanism. This mechanism is a mathematical representation of a decision procedure where changes of all agents' statements about the desired outcome are taken into account. It is one example of a mechanism where the intensity of an individual's statement always affects the collective decision. Let

$$x_{Mean} = x_{Mean}(\hat{\theta}) := \frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i. \tag{6}$$

This mechanism asks the individuals for their private information and implements the average of all announcements. Consequently, the mean mechanism uses all available information, changes in extreme positions are taken into account and thus extreme positions influence the common decision. This mechanism would implement the welfare maximizing solution if agents were to tell the truth. Our analysis of the mean mechanism therefore describes the outcome when people behave rational in a set-up where truth-telling would maximize welfare.

3 The Results

In this section, we present two Bayesian Nash equilibria of the games introduced above, one of the median and one of the mean mechanism. These equilibria imply truthful revelation for certain degrees of interdependencies.

[5]Let θ_{Median} be the signal of the median individual. In general $\theta_{Median} = \hat{\theta}_{Median}$ is not true.

3.1 Equilibria

It is useful to define the desired policy of type θ_i conditional on being the median individual. It is given by

$$x(\theta_i) := (1-\alpha)\,\theta_i + \alpha\,\left(\frac{1}{2}E\,\left[\theta_k \mid \theta_k < \theta_i\right] + \frac{1}{2}E\,\left[\theta_k \mid \theta_k > \theta_i\right]\right). \tag{7}$$

This is the expected value of player $i's$ ideal point θ_i^* conditional on player i being the median individual. We show that the profile of announcements $\widehat{\theta} = \left(\widehat{\theta}_1, ..., \widehat{\theta}_n\right)$ where $\hat{\theta}_i = x(\theta_i)\ \forall i = 1, .., n$ constitutes a symmetric Bayesian Nash equilibrium under the median mechanism. There is one and only one such equilibrium.[6]

Let $U\left(\hat{\theta}_i, \theta_i\right)$ denote the expected utility of player i when he is of type θ_i, announces $\hat{\theta}_i$ and all players $j \neq i$ announce according to $x(\theta_j)$. In a first step, we show that playing $x(\theta_i)$ is locally optimal when all other players play $x(\theta_j)$.

Lemma 2 *For given θ_i, the function $U\left(\hat{\theta}_i, \theta_i\right)$ has a local maximum at $x(\theta_i)$.*

Proof of Lemma 2:

Consider a marginal deviation from this announcement $x(\theta_i)$. Either player i is pivotal or he is not pivotal when he announces $x(\theta_i)$. If player i is not pivotal then a marginal deviation does not alter the outcome x_{Median}. If player i is pivotal, then his announcement is the median announcement and therefore his type θ_i is the median of all types. A marginal deviation does not pay because it alters the implemented decision x_{Median}. However, $x(\theta_i)$ is already chosen optimally conditional on player i being the median. Hence, any marginal deviation from $x(\theta_i)$ reduces $U\left(\hat{\theta}_i, \theta_i\right)$. Q.E.D.

[6] There exists a continuum of other symmetric equilibria in which all individuals announce the same type irrespective of their signal, i.e. $\hat{\theta}_i(\theta_i) = \widetilde{\theta}\ \forall i$. If all others announce according to $\hat{\theta}_j(\theta_j) = \widetilde{\theta}$, it is a best reply for i to announce $\widetilde{\theta}$ as well, since on no account its announcement will change the implemented decision under the median mechanism.

Lemma 3 *For given* θ_i*, the function* $U\left(\hat{\theta}_i, \theta_i\right)$ *is single peaked at* $x(\theta_i)$*.*

Proof of Lemma 3:

Consider any other announcement $\hat{\theta}_i' < x(\theta_i)$ of player i. Either player i is pivotal at $\hat{\theta}_i'$ or he is not pivotal. If player i is not pivotal, then a marginal deviation does not alter the outcome x_{Median}. If his announcement is pivotal, then one half of the other players $j \neq i$ makes an announcement below $\hat{\theta}_i'$. This in turn means that one half of the other players has a type below $x^{-1}\left(\hat{\theta}_i'\right)$, since all $j \neq i$ announce according to $x(\theta_j)$.[7] Player $i's$ conditional desired policy is then given by the expected ideal point of player i conditional on one half of the other players having types smaller than $x^{-1}\left(\hat{\theta}_i'\right)$, i.e.

$$x'(\theta_i) = (1-\alpha)\,\theta_i + \alpha\left(\frac{1}{2}E\left[\theta_k \mid \theta_k < x^{-1}\left(\hat{\theta}_i'\right)\right] + \frac{1}{2}E\left[\theta_k \mid \theta_k > x^{-1}\left(\hat{\theta}_i'\right)\right]\right). \quad (8)$$

On the other hand, according to the definition of $x(\theta_i)$ it must be true that

$$\hat{\theta}_i' = (1-\alpha)\,x^{-1}\left(\hat{\theta}_i'\right) + \alpha\left(\frac{1}{2}E\left[\theta_k|\theta_k < x^{-1}\left(\hat{\theta}_i'\right)\right] + \frac{1}{2}E\left[\theta_k|\theta_k > x^{-1}\left(\hat{\theta}_i'\right)\right]\right). \quad (9)$$

Finally, it follows from

$$\hat{\theta}_i' < x(\theta_i) \Leftrightarrow x^{-1}\left(\hat{\theta}_i'\right) < \theta_i \quad (10)$$

and from equations (8) and (9) that

$$\hat{\theta}_i' < x'(\theta_i). \quad (11)$$

Thus, a marginal increase in the announcement $\hat{\theta}_i'$ increases individual $i's$ expected utility, since it shifts the implemented decision in the direction of its ideal point.

[7] Note that $x^{-1}(\cdot)$ exists because the expected signal of the others conditional on being the median type is strictly increasing in the type if the distribution has full support. Hence, $x(\cdot)$ is strictly increasing in θ_i. Moreover, $x(\cdot)$ is differentiable in θ_i.

The same line of argumentation holds for announcements $\hat{\theta}_i' > x(\theta_i)$. The single peakedness of $U\left(\hat{\theta}_i, \theta_i\right)$ at $x\left(\theta_i\right)$ follows from this and from the local optimality of $x(\theta_i)$. Q.E.D.

Proposition 1 *There exists a unique strictly monotonous symmetric equilibrium under the median mechanism. The equilibrium strategy is*

$$x(\theta_i) = (1-\alpha)\,\theta_i + \alpha\,\left(\frac{1}{2}E\left[\theta_k \mid \theta_k < \theta_i\right] + \frac{1}{2}E\,\left[\theta_k \mid \theta_k > \theta_i\right]\right)\ \forall i = 1, ..., n.$$

Proof of Proposition 1:

Existence and uniqueness follow immediately from Lemmata 2 and 3. Q.E.D.

In the special case of a uniform distribution the equilibrium strategy turns out to be linear.

Corollary 1 *Assume that all θ_i are distributed uniformly over $[-1, 1]$. Then, the equilibrium strategy under the median mechanism is given by $\hat{\theta}_i(\theta_i) = \left(1 - \frac{1}{2}\alpha\right)\theta_i\ \forall i$.*

Under the median mechanism individuals understate their private information. With increasing degrees of interdependency (larger α) the private signal becomes less valuable. Since individuals know that the median of all announcements will be implemented, they try to profit from the others' information. Their announcement is closer to zero than their true signal. This is due to the fact that individuals with extreme signals take into account that - if they are the median - the expected value of the other agents' signals is closer to zero than their own signal.

While the median mechanism has an equilibrium in linear strategies only for a certain distribution of types, the mean mechanism always has a linear equilibrium. This equilibrium is unique up to a constant. According to this strategy individuals overstate their private information.

Proposition 2 *For all constants $c_1, ..., c_n$ with $\sum_{i=1}^{n} c_i = 0$ the following strategies constitute a Bayesian Nash equilibrium: $\hat{\theta}_i(\theta_i) = n(1-\alpha)\theta_i + c_i\ \forall i$. There exists no other equilibrium.*

Proof of Proposition 2:

Take any profile $\hat{\theta}_{-i} = \left(\hat{\theta}_1(\theta_1), ..., \hat{\theta}_{i-1}(\theta_{i-1}), \hat{\theta}_{i+1}(\theta_{i+1}), ..., \hat{\theta}_n(\theta_n)\right)$ as given. Individual i maximizes its expected utility if $\hat{\theta}_i$ maximizes

$$E\left[-(x_{Mean} - \theta_i^*)^2\right] = E\left[-\left(\frac{\sum_{j=1}^{n}\hat{\theta}_j}{n} - \theta_i^*\right)^2\right]. \tag{12}$$

Substituting for θ_i^* yields

$$\max_{\hat{\theta}_i} E\left[-\left(\frac{\sum_{j=1}^{n}\hat{\theta}_j}{n} - (1-\alpha)\theta_i - \frac{\alpha}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\theta_j\right)^2\right] \tag{13}$$

or

$$\max_{\hat{\theta}_i} E\left[-\left(\frac{\hat{\theta}_i}{n} + \frac{1}{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\hat{\theta}_j - (1-\alpha)\theta_i - \frac{\alpha}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\theta_j\right)^2\right]. \tag{14}$$

Let

$$a := \frac{1}{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\hat{\theta}_j - (1-\alpha)\theta_i - \frac{\alpha}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\theta_j. \tag{15}$$

Then, (14) becomes

$$\max_{\hat{\theta}_i} E\left[-\left(\frac{\hat{\theta}_i}{n} + a\right)^2\right] \tag{16}$$

or

$$\max_{\hat{\theta}_i} E\left[-\frac{\hat{\theta}_i^2}{n^2} - \frac{2a}{n}\hat{\theta}_i - a^2\right]. \tag{17}$$

Optimality requires

$$-\frac{2}{n^2}\hat{\theta}_i - \frac{2}{n}E[a] = 0 \tag{18}$$

and thus,

$$\hat{\theta}_i = -nE\left[a\right]. \tag{19}$$

Using the fact that all θ_j have an expected value of zero and that expectations are taken over all $j \neq i$, we get

$$E\left[a\right] = \frac{1}{n}E\left[\sum_{\substack{j=1\\j\neq i}}^{n}\hat{\theta}_j\right] - \left(1-\alpha\right)\theta_i \tag{20}$$

and thus,

$$\hat{\theta}_i = n\left(1-\alpha\right)\theta_i - E\left[\sum_{\substack{j=1\\j\neq i}}^{n}\hat{\theta}_j\right]. \tag{21}$$

Hence, the best reply with respect to any profile $\hat{\theta}_{-i}$ has to be linear with slope $n\left(1-\alpha\right)$. The fact that $\sum_{i=1}^{n} c_i = 0$ follows from (21) and the assumption that $E\left[\theta_i\right] = 0\ \forall i$. Q.E.D.

In what follows we restrict attention to announcement strategies of the form $\hat{\theta}_i(\theta_i) = n\left(1-\alpha\right)\theta_i\ \forall i$, since the constant c_i has no effect on expected utility.

Under the mean mechanism individuals exaggerate their private signal in order to cancel out the average taking implied by this mechanism. With increasing degrees of interdependency this behavior becomes counterproductive and announcements approach the true signal.

Using the above calculated equilibrium strategies, it follows that

Corollary 2 *The median mechanism has a symmetric Bayesian Nash equilibrium in which agents announce their type truthfully if* $\alpha = 0$.

Corollary 3 *The mean mechanism has a symmetric Bayesian Nash equilibrium in which agents announce their type truthfully if* $\alpha = \bar{\alpha} = \frac{n-1}{n}$.

4 Comparison of mean and median mechanism

This section compares the performance of the median and the mean mechanism for different degrees of interdependencies. First, we present results that obtain independent of the underlying distribution of information parameters and for any number of agents. In the second part, we develop a concrete example with uniform distribution and a small number of individuals.

4.1 General results

We start the comparison by analyzing the properties of the two mechanisms at the corner values of α, i.e. at $\alpha = 0$ and at $\alpha = \bar{\alpha}$.

Proposition 3 *For $\alpha = 0$ and $n \geq 3$ the sum of expected utilities is larger under the median than under the mean mechanism.*

Proof of Proposition 3:

For $\alpha = 0$ it holds that $\theta_i^* = \theta_i$. Since the whole setting is symmetric, it suffices to show that individual expected utility is larger under the median than under the mean mechanism, i.e. that

$$E\left[-\left(x_{Median} - \theta_i\right)^2\right] > E\left[-\left(x_{Mean} - \theta_i\right)^2\right]. \tag{22}$$

First, consider the mean mechanism. In equilibrium individuals announce according to $\widehat{\theta}_i\left(\theta_i\right) = n\theta_i$ and thus expected utility for individual i is given by

$$\begin{aligned} E\left[-\left(x_{Mean} - \theta_i\right)^2\right] &= E\left[-\left(\frac{\sum_{i=1}^{n} n\,\theta_i}{n} - \theta_i\right)^2\right] \\ &= E\left[-\left(\sum_{i=1}^{n} \theta_i - \theta_i\right)^2\right] \end{aligned}$$

$$
\begin{aligned}
&= E\left[-\left(\sum_{\substack{j=1\\ j\neq i}}^{n}\theta_j\right)^2\right] \\
&= -(n-1)\,E\left[\theta_i^2\right]. && (23)
\end{aligned}
$$

Under the median mechanism individuals announce their type truthfully, i.e. $\widehat{\theta}_i(\theta_i) = \theta_i$. Let θ_{Median} denote the signal of the median individual. Consider a situation in which n individuals decide and individual i is not included in the collective decision, i.e. a situation in which θ_{Median} and θ_i are not correlated. Expected utility in this situation represents an upper bound to losses (a lower bound to individual expected utility), since participating in the decision process can only improve upon the situation. If θ_{Median} and θ_i are independent, individual expected utility is given by

$$
\begin{aligned}
E\left[-\left(x_{Median}-\theta_i\right)^2\right] &= E\left[-\left(\theta_{Median}-\theta_i\right)^2\right] && (24)\\
&= -E\left[\theta_{Median}^2\right]-E\left[\theta_i^2\right] \\
&> -2E\left[\theta_i^2\right]. && (25)
\end{aligned}
$$

The last inequality follows, because $E\left[\theta_{Median}^2\right]$ can never exceed the variance of any single information parameter θ_i. This is true because the product of probabilities can never exceed 1:

$$
\begin{aligned}
& E\left[\theta_{Median}^2\right] && (26)\\
=\ & \int_{\Theta}\theta_i^2\, p\left[\theta_1\leq\theta_i\right]...p\left[\theta_{\frac{n-1}{2}}\leq\theta_i\right]\ p\left[\theta_{\frac{n-1}{2}+1}\geq\theta_i\right]...p\left[\theta_n\geq\theta_i\right]\ f(\theta_i)d\theta_i \\
<\ & \int_{\Theta}\theta_i^2 f(\theta_i)d\theta_i \\
=\ & E\left[\theta_i^2\right].
\end{aligned}
$$

It remains to show that the lower bound on expected utility under the median exceeds expected utility under the mean mechanism for $n \geq 3$:

$$-2E\left[\theta_i^2\right] > -(n-1)\,E\left[\theta_i^2\right] \tag{27}$$

$\Leftrightarrow$

$$(n-3)\,E\left[\theta_i^2\right] > 0 \tag{28}$$

which is true for all $n > 3$. For $n = 3$ the lower bound on expected utility under the median equals expected utility under the mean mechanism. Since the lower bound is strict, this completes the proof. Q.E.D.

The median mechanism induces agents to tell the truth in the private values case. This leads to a relatively high welfare level when median and mean are correlated. The mean mechanism instead induces all agents to exaggerate their private information. This explains the superiority of the median mechanism. Note that the median mechanism does not implement the first-best in general, since the first-best would only be implemented if the private information of the median individual by instance is equal to the average of all types.

Proposition 4 *For $\alpha = \bar{\alpha}$ it holds that (i) the mean mechanism yields the first-best and (ii) the median mechanism does not yield the first-best.*

PROOF OF PROPOSITION 4:

(i) From Proposition 2 it follows that under the mean mechanism agents announce their type truthfully if $\alpha = \bar{\alpha}$. Thus $x_{Mean} = \frac{1}{n}\sum_{i=1}^{n}\theta_i$ and it holds that $\theta_i^* = \frac{1}{n}\sum_{i=1}^{n}\theta_i$. We obtain for the sum of expected utilities

$$E\left[\sum_{i=1}^{n} -(x_{Mean} - \theta_i^*)^2\right] = 0. \tag{29}$$

(ii) It remains to show that

$$0 > E\left[\sum_{i=1}^{n} - \left(x_{Median} - \theta_i^*\right)^2\right]. \tag{30}$$

Suppose that the equilibrium under the median mechanism from Proposition 1 maximizes expected utility. Let $s(\theta)$ be the respective equilibrium profile. Then

$$x\left(s\left(\theta\right)\right) = \theta_{Mean} = \frac{1}{n}\sum_{i=1}^{n}\theta_i \; \forall \; \theta. \tag{31}$$

However, from Proposition 1 we know that for some θ some partial derivatives of $x\left(s\left(\theta\right)\right)$ are zero. This is so because the median is unaffected by changes in extreme positions. Hence, $x\left(s\left(\theta\right)\right)$ cannot equal θ_{Mean} for all θ. Q.E.D.

The intuition for this result is as follows: for $\alpha = \bar{\alpha}$ individuals announce their type truthfully under the mean mechanism and we know already that the first-best solution is the average of all types. This yields part (i). However, the median mechanism does neither imply truth-telling for $\alpha = \bar{\alpha}$ nor does it implement the average of all private signals. Therefore, it does not yield the first-best.

Now, we turn to the behavior of the sum of expected utilities under both mechanisms for intermediate values of α.

Lemma 4 *Consider the median mechanism. The sum of expected utilities is continuous in α.*

Proof: See Appendix.

Lemma 5 *Consider the mean mechanism. The sum of expected utilities is (i) continuous in α, (ii) strictly increasing in α and (iii) attains its maximum at $\alpha = \bar{\alpha}$.*

Proof: See Appendix.

Lemma 6 *Consider the mean mechanism. For $n > 1$ the sum of expected utilities is (i) continuous in n, (ii) strictly decreasing in n and (iii) attains its maximum at $n = \frac{1}{1-\alpha}$.*

Proof: See Appendix.

All the above yields our main result concerning the comparison between the median and the mean mechanism:

Proposition 5 *There exists an α_1 below which the sum of expected utilities is larger under the median than under the mean mechanism and there exists an α_2 above which the sum of expected utilities is larger under the mean than under the median mechanism.*

Proof of Proposition 5:

The result follows directly from Propositions 3 and 4 and Lemmata 4 and 5. Q.E.D.

If individual preferences are strongly correlated, then making all agents participate in the decision is better than restricting entry into the decision process. If there is only a small common component then it is better to use the median mechanism. The intuition is that for weak interdependencies the equilibrium strategy under the median mechanism implies announcement behavior close to truth-telling whereas the equilibrium strategy under the mean mechanism leads to strong exaggeration of private information. Therefore, average taking is outperformed by ignoring some of the information available. Since the degree to which α influences untruthful announcement behavior is stronger under the mean mechanism, this intuition holds for a wide range of interdependencies, only for very high degrees it is reversed.

4.2 Example with uniform distribution

Assume that all θ_i are distributed uniformly over $[-1,1]$ and that $n=3$.[8] Then, the comparison between the median and the mean mechanism can be extended and we obtain the result that there exists a unique point of intersection for the respective sums of expected utility under the two mechanisms.

Corollary 1 tells us that for uniform distribution the equilibrium strategy under the median mechanism is given by $\hat{\theta}_i(\theta_i) = \left(1-\frac{1}{2}\alpha\right)\theta_i \;\forall i$ and thus $x_{Median} = (1-\frac{1}{2}\alpha)\theta_{Median}$. Then, expected utility for individual 2 is given by

$$\begin{aligned}
& E\left[-\left(x_{Median}-\theta_2^*\right)^2\right] \\
= & -\frac{1}{4}\int_{-1}^{1}\int_{-1}^{\theta_2}\int_{\theta_2}^{1}\left(\left(1-\frac{1}{2}\alpha\right)\theta_2-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
& -\frac{1}{4}\int_{-1}^{1}\int_{-1}^{\theta_2}\int_{\theta_1}^{\theta_2}\left(\left(1-\frac{1}{2}\alpha\right)\theta_3-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
& -\frac{1}{4}\int_{-1}^{1}\int_{\theta_2}^{1}\int_{\theta_1}^{1}\left(\left(1-\frac{1}{2}\alpha\right)\theta_1-(1-\alpha)\,\theta_2-\frac{\alpha}{2}\left(\theta_1+\theta_3\right)\right)^2 d\theta_3 d\theta_1 d\theta_2 \\
= & -\frac{11}{20}\alpha^2+\frac{11}{15}\alpha-\frac{4}{15}. \qquad (32)
\end{aligned}$$

Since our whole setting is symmetric, we obtain for the sum of expected utilities under the median mechanism

$$\sum_{i=1}^{3} E\left[-\left(x_{Median}-\theta_2^*\right)^2\right] = -\frac{33}{20}\alpha^2+\frac{11}{5}\alpha-\frac{4}{5}. \qquad (33)$$

Differentiating with respect to α yields

$$\frac{d\left(E\left[\sum_{i=1}^{3}-(x_{Median}-\theta_i^*)^2\right]\right)}{d\alpha} = -\frac{33}{10}\alpha+\frac{11}{5} \qquad (34)$$

which is larger than zero $\forall\ \alpha \in [0,\bar{\alpha})$. Thus, the sum of expected utilities is strictly increasing under the median mechanism and yields a maximum for $\alpha = \frac{2}{3}$.

[8] We conducted a similar analysis for the case of $n=5$ and $n=7$, but for expositional purposes only the detailed results for $n=3$ are given.

Under the mean mechanism agents announce $\hat{\theta}_i(\theta_i) = 3(1-\alpha)\theta_i$ for $n = 3$. Thus, $x_{Mean} = (1-\alpha)\sum_{i=1}^{3}\theta_i$. The sum of expected utilities is then given by

$$\begin{aligned}
& \sum_{i=1}^{3} E\left[-\left(x_{Mean} - \theta_i^*\right)^2\right] \\
= & -\frac{3}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\left((1-\alpha)\sum_{i=1}^{3}\theta_i - (1-\alpha)\theta_1 - \frac{\alpha}{2}(\theta_2+\theta_3)\right)^2 d\theta_1 d\theta_2 d\theta_3 \\
= & -\frac{9}{2}\alpha^2 + 6\alpha - 2. \qquad (35)
\end{aligned}$$

Differentiating with respect to α yields

$$\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean} - \theta_i^*)^2\right]\right)}{d\alpha} = -9\alpha + 6. \qquad (36)$$

From Proposition 5 we know that there exists an α_1 below which the sum of expected utilities is larger under the median than under the mean mechanism and that there exists an α_2 above which the sum of expected utilities is larger under the mean than under the median mechanism. For uniform distribution of the information parameters the two points coincide, i.e. $\alpha_1 = \alpha_2 =: \alpha^*$.

The existence of an unique point of intersection can be established by comparing the slopes of the sum of expected utility under the two mechanisms, since both are strictly positive for the relevant range of α. Under the median mechanism the slope is given by equation (34) and under the mean mechanism by equation (36), respectively. For all $\alpha \in [0, \bar{\alpha})$ it is true that

$$-\frac{33}{10}\alpha + \frac{11}{5} < -9\alpha + 6 \qquad (37)$$

and therefore α^* exists.

The sum of expected utilities under the median mechanism is given by equation (33) and under the mean mechanism by equation (35). The point of intersection α^*

is then determined by

$$-\frac{33}{20}\alpha^{*2} + \frac{11}{5}\alpha^* - \frac{4}{5} = -\frac{9}{2}\alpha^{*2} + 6\alpha^* - 2 \tag{38}$$

which yields

$$\alpha^*_{1,2} = \frac{2}{3} \pm \frac{2}{57}\sqrt{19}. \tag{39}$$

Since $\alpha^*_1 > \bar{\alpha} = \frac{2}{3}$, α^*_2 is the unique point of intersection in the relevant interval.

The following pictures show the sum of expected utilities under the mean and the median mechanism for $n = 3$, 5 and 7 and uniform distribution of information parameters. In each of these cases, we obtain a unique point of intersection α^*.

Figure 1: $n = 3$

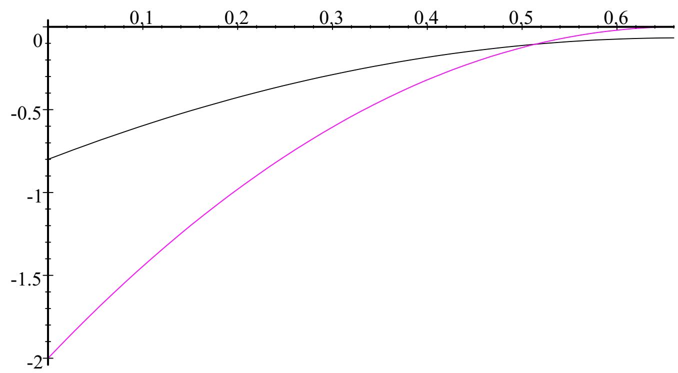


Figure 2: $n = 5$

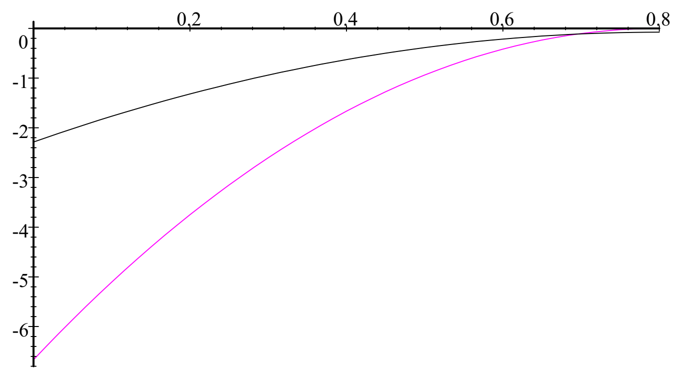
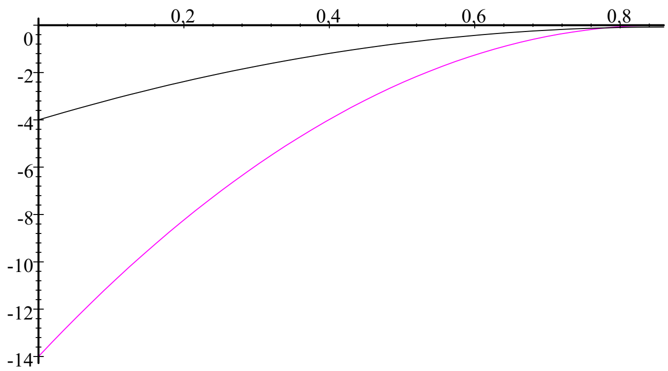


Figure 3: $n = 7$



5 Robustness

So far, our results have been derived for a particular class of quadratic utility functions. These utility functions describe a situation where individuals are risk averse with respect to the size of the deviation of the decision x from their ideal point θ_i^*. Our result on the welfare comparison of the two mechanisms presented in Proposition 5 also holds under a linear specification of utility which corresponds to the case of risk neutrality.

Consider the following von-Neumann-Morgenstern utility function

$$u(x, \theta_i^*) = -\left|x - \theta_i^*\right|. \tag{40}$$

In contrast to Lemma 1 we get

Lemma 7 *Consider any vector θ of private signals. Benthamian welfare is maximized if and only if $x^* = median(\theta_1^*, ..., \theta_n^*)$.*

Proof of Lemma 7:

Consider any deviation from the median of all ideal policies, i.e. from $median(\theta_1^*, ..., \theta_n^*)$. With linear utility, this deviation costs more than half of the individuals one unit while less than half of the individuals gain. Q.E.D.

Figure 4 presents the basic intuition underlying this result for the case of three individuals. This extends to any number n of individuals.

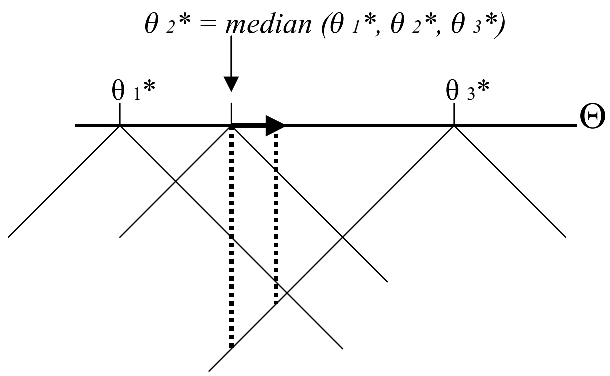


Figure 4

It is useful to consider the implications of the changed utility specification for an individually desired policy $\widetilde{x}$ when individual i does not know its ideal point with certainty.[9] Since all θ_i are distributed independently with density $f(\theta_i)$, this defines a probability distribution for each θ_i^* over Θ. Let $\psi(\theta_i^*)$ denote the density of this distribution and $median(\psi(\theta_i^*))$ the respective median.

Lemma 8 *Consider an individual i with an ideal point θ_i^* distributed according to the density $\psi(\theta_i^*)$. Ex-ante individual i's preferred decision $\widetilde{x}$ is the median of the distribution of his own ideal points, i.e. median $(\psi(\theta_i^*))$.*

[9] Under quadratic specification of utility this policy $\widetilde{x}$ corresponds to the mean of the distribution of the individually desired policy θ_i^*.

Proof of Lemma 8:

Any deviation from this choice triggers a marginal loss of 1 with probability of more than 1/2 and a marginal gain of one unit with a probability below 1/2. Q.E.D.

Proposition 1 no longer holds under the linear specification of utility. In order to transfer the result, we have to introduce some more notation: Let $\phi(\theta_i)$ denote the density of the distribution of the average of the signals of all agents $j \neq i$ conditional on θ_i being the median. Let $median(\phi(\theta_i))$ denote the median of this distribution.

Proposition 6 *Consider a linear utility function of the form $u(x, \theta_i^*) = -|x - \theta_i^*|$. There exists a unique symmetric strictly monotonous equilibrium under the median mechanism. The equilibrium strategy is*

$$\hat{\theta}_i(\theta_i) = (1-\alpha)\,\theta_i + \alpha\; median\,(\phi(\theta_i)) \;\; \forall i, i = 1, ..., n. \tag{41}$$

Proof of Proposition 6:

Following the same line of argumentation that leads to Proposition 1, note first that the announcement $\hat{\theta}_i(\theta_i)$ is a local maximum of player $i's$ expected utility given θ_i and all other $j \neq i$ announcing according to $\hat{\theta}_j(\theta_j)$. Consider a marginal deviation from this announcement. If player i is not pivotal, then a marginal deviation does not alter the outcome x_{Median}. If player i is pivotal, then we know from Lemma 8 that he chooses the median of the distribution of his own ideal points as the common decision. Conditional on being the median, this becomes $(1-\alpha)\,\theta_i + \alpha\; median\,(\phi(\theta_i))$. Thus, marginal deviations from $\hat{\theta}_i(\theta_i)$ reduce expected utility of player i.

Moreover, player $i's$ expected utility is single peaked at $\hat{\theta}_i(\theta_i)$ given θ_i and all other $j \neq i$ announcing according to $\hat{\theta}_j(\theta_j)$. This is due to the monotonicity of the median of $\phi(\theta_i)$ in the signal of player i and the consequential existence of $\hat{\theta}_i^{-1}(\cdot)$. This completes the proof. Q.E.D.

It can be easily seen that the equilibrium of Proposition 2 still exists under the linear specification of the utility function if the distribution of information parameters is symmetric. The intuition is that any change in θ_i does neither affect the median of the distribution of the average of the signals of the other individuals nor does it affect the median of the distribution of the average of the announcements of the other individuals. Therefore, according to Lemma 8, agent $i's$ favorite decision is shifted by $n(1-\alpha)\Delta\theta_i$.

Another straightforward consequence of Lemma 7 is that the median mechanism maximizes ex-ante expected utility in the private values case. In this setting, the median mechanism (again) induces truth-telling. This maximizes welfare with linear utility. Moreover, for symmetric distributions of types, the median mechanism still yields higher ex-ante expected utility than the mean mechanism in the private values case. Proposition 4 (i) on the mean mechanism instead holds and it remains true that the mean yields more expected utility than the median mechanism in the case where $\alpha = \overline{\alpha}$. Continuity again implies that there exist upper and lower bounds on α such as in Proposition 5.

6 Conclusion

The problem analyzed in this paper is one of collective decision taking. If the individuals who are supposed to take a common decision are asymmetrically informed and if there are interdependencies between the individually desired policies, how should a decision mechanism be designed that maximizes the sum of expected utilities? Our analysis of this problem concentrates on two specific mechanisms, i.e. the median and the mean mechanism, and obtains the following: Under the median mechanism individuals understate their private information, whereas under the mean mechanism they overstate it. As a result, the median performs better than the mean mechanism in terms of expected utility maximization for weak interdependencies. For high degrees of interdependency is this mechanism outperformed by the mean mechanism.

Returning to the provision of a public good, we can conclude that the median mechanism performs better than taking the average if the idiosyncratic component of individual preferences is strong. Disregarding extreme positions - as does the median mechanism - seems to be favorable in this case. Only if the quality component becomes very important, it would be better to change the decision mechanism and use the average of announcements.

Starting from our research there are three directions to proceed. First, in this paper we abstracted from individual rationality considerations, since in many collective decisions participation is not voluntary. However, if we take participation constraints into account, the traditional solution prescribes an outside option to be implemented if an individual opts out. This in turn leads to changed interim individual behavior and to different equilibrium outcomes - with the status quo maintaining in many instances. But in our setting, due to interdependent valuations, even individuals not participating in the mechanism would be affected by the common decision. This would imply endogenizing the participation constraint (following Jehiel et al.[1996]).

Relating our results to decisions taken by majority rule, it is not obvious how a majoritarian decision in a committee should be modeled. In the private values case the median mechanism is certainly a good approximation of the actual decision procedure. However, in a setting with interdependencies there may be scope for pre-vote communication. When communication plays a role, the median mechanism need not be the correct representation of a majoritarian decision. However, the result of our paper indicates, that voting without communication can dominate the mean mechanism. This leads to the second extension, the analysis of pre-vote communication in a modified two-stage game. The question is if an improvement upon the equilibria of the original game is possible when people are allowed to communicate before they have to vote. It is well known that equilibrium behavior can be affected if agents have the opportunity to exchange information prior to playing some game (see Crawford and Sobel [1982]).

Finally, another question we did not address is the design of an optimal mechanism for the class of collective decision problems studied. This would mean to find a mechanism that implements the first-best for all degrees of spillovers, not only for the maximum amount.

7 Appendix - Proofs

PROOF OF LEMMA 4:

Under the median mechanism the announcement strategies $\hat{\theta}_i(\theta_i)$ are continuous in the parameter α. Since the decision $x_{Median}(\hat{\theta})$ is continuous in the announcements, it is so in α. Finally, individual utility is continuous in the decision (and therefore in α) and in the individually desired policy θ_i^*, which is continuous in α as well. Thus the sum of expected utilities is continuous in α under the median mechanism. Q.E.D.

PROOF OF LEMMA 5:

Under the mean mechanism agents announce according to $\hat{\theta}_i(\theta_i) = n(1-\alpha)\theta_i$. Thus, $x_{Mean} = (1-\alpha)\sum_{i=1}^n \theta_i$.

(i) CONTINUITY: The sum of expected utilities is given by

$$\begin{aligned}
\sum_{i=1}^n E\left[-\left(x_{Mean}-\theta_i^*\right)^2\right] &= -nE\left[\left((1-\alpha)\sum_{i=1}^n \theta_i - (1-\alpha)\theta_i - \frac{\alpha}{n-1}\sum_{j\neq i}\theta_j\right)^2\right] \\
&= -nE\left[\left((1-\alpha)\sum_{\substack{j=1\\ j\neq i}}^n \theta_j - \frac{\alpha}{n-1}\sum_{\substack{j=1\\ j\neq i}}^n \theta_j\right)^2\right] \\
&= -nE\left[\left(-\frac{\alpha n - n + 1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^n \theta_j\right)^2\right] \\
&= -n\left(\frac{\alpha n - n + 1}{n-1}\right)^2 E\left[\left(\sum_{\substack{j=1\\ j\neq i}}^n \theta_j\right)^2\right] \qquad (42)
\end{aligned}$$

which is continuous in α.

(*ii*) Monotonicity: Differentiating with respect to α yields

$$\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean}-\theta_i^*)^2\right]\right)}{d\alpha} = -2n^2\frac{\alpha n-n+1}{(n-1)^2}E\left[\left(\sum_{\substack{j=1\\j\neq i}}^{n}\theta_j\right)^2\right] \tag{43}$$

which is larger than zero $\forall\ \alpha \in [0,\bar{\alpha})$.

(*iii*) Optimality:

$$\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean}-\theta_i^*)^2\right]\right)}{d\alpha} = 0 \Longleftrightarrow \alpha = \frac{n-1}{n} = \bar{\alpha}\ . \tag{44}$$

Q.E.D.

Proof of Lemma 6:

(*i*) Continuity: The sum of expected utilities under the mean mechanism is given by

$$\begin{aligned}\sum_{i=1}^{n} E\left[-\left(x_{Mean}-\theta_i^*\right)^2\right] &= -n\left(\frac{\alpha n-n+1}{n-1}\right)^2 E\left[\left(\sum_{\substack{j=1\\j\neq i}}^{n}\theta_j\right)^2\right]\\ &= -n\left(n-1\right)\left(\frac{\alpha n-n+1}{n-1}\right)^2 E\left[\theta_i^2\right]\end{aligned} \tag{45}$$

which is continuous in n for $n>1$.

(*ii*) Monotonicity: Differentiating with respect to n yields

$$\begin{aligned}&\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean}-\theta_i^*)^2\right]\right)}{dn}\\ = &-\left(\alpha n-n+1\right)\frac{2\alpha n^2-3\alpha n-2n^2+3n-1}{(n-1)^2}E\left[\theta_i^2\right].\end{aligned} \tag{46}$$

Since $\alpha n-n+1<0$ and $2\alpha n^2-3\alpha n-2n^2+3n-1<0\ \forall\ \alpha\in[0,\bar{\alpha})$, this is smaller than zero $\forall\ n>1$.

(*iii*) Optimality:

$$\frac{d\left(E\left[\sum_{i=1}^{n} -(x_{Mean}-\theta_i^*)^2\right]\right)}{dn} = 0 \Longleftrightarrow n = \frac{1}{1-\alpha}. \tag{47}$$

Q.E.D.

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