Cutting out the Middleman: Crowdinvesting, Efficiency, and Inequality – online appendix –

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Abstract

We present an efficient equilibrium of the dynamic investment model, which is introduced in the robustness section of the main paper, in more detail.

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Sequential investments

Consider the following modification of the baseline setup from section 2 of the paper. In $t = 0$, all crowdinvestors may condition their investment plans $\hat{x}_i^{t=0}(\theta_i)$ only on their own private information, leading to aggregate investment $X_0 = \int_0^1 \hat{x}_i^0(\theta_i)di$. In $t = 1$, all crowdinvestors may condition their investment plans $\hat{x}_i^1(\theta_i, X_0)$ on their private information and aggregate investment from the previous investment stage. In equilibrium, investors can adjust their investment to the realization of X_0, and thereby use the information contained in X_0 about the distribution of θ_i when investing at $t = 1$. Overall investment by crowdinvestor i in the company is $\hat{x}_i^0(\theta_i) + \hat{x}_i^1(\theta_i, X_0)$, i.e., the sum of the investments in $t = 0$ and $t = 1$, with $\hat{x}_i^t \geq 0$ as before. The following equilibrium definition extends the static baseline model to a two stage investment game.

Definition 3. *An equilibrium of the dynamic model consists of*

i. a consumption plan $x_i(p)$ for each consumer,

ii. an investment plan $\hat{x}_i^0(\theta_i)$ at $t = 0$ and an investment plan $\hat{x}_i^1(\theta_i, X_0)$ at $t = 1$ for each consumer,

iii. a relative price function $p(X, s)$ for good x,

such that

i. the consumption plan maximizes utility subject to the consumer's period 2 budget constraint,

ii. the investment plans constitute a Bayesian Nash equilibrium of the investment game subject to the wealth constraints, taking into account the consumption plans and the relative price $p(X, s)$, and

iii. at price $p(X, s)$ the aggregate demand for good x equals supply x_{sup}.

The following proposition illustrates an efficient equilibrium if consumers of the poor group do not have enough wealth to finance the efficient individual investment $(\alpha/R)^{1/(1-\alpha)}$ on their own. Efficient investment is achieved because the wealthy group reacts to early investments—which depend on the preference distribution—by the poor group in $t = 0$.

Proposition 1. *Suppose w_i is constant within each group. Then there exists an efficient equilibrium if wealth $\underline{w}$ in the poor group 2 is positive but insufficient to finance the individual efficient investment,*

$$(\alpha/R)^{1/(1-\alpha)} > \underline{w} = w_i > 0 \ \forall i \in (0.5, 1],$$

and if wealth $\overline{w}$ in the wealthy group 1 fulfills

$$\overline{w} = w_i \geq \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right) \beta/2 + (\alpha/R)^{1/(1-\alpha)} \ \forall i \in [0, 0.5].$$

Proof. By construction. Suppose we have an equilibrium with the following $t = 0$ investment strategy profile:

$$\hat{x}_i^0(\theta_i = 1) = \underline{w} \ \forall i \in (0.5, 1],$$
$$\hat{x}_i^0(\theta_i = 0) = 0 \ \forall i \in (0.5, 1],$$
$$\hat{x}_i^0(\theta_i) = 0 \ \forall i \in [0, 0.5],$$

resulting in aggregate investment $X_0(s_2) = \underline{w}s_2/2$, which is strictly increasing in the realization of $s_2 \in \{1-\beta, \beta\}$. Consequently, on the equilibrium path, we can write the realization of s_2 implied by the observed aggregate investment x_0 as the inverse of the equilibrium $t = 0$ aggregate investment function: $s_2 = X_0^{-1}(X_0 = x_0)$. Consider now the following $t = 1$ investment strategy profile:

$$\hat{x}_i^1(\theta_i, X_0) = 0 \ \forall i \in (0.5, 1],$$
$$\hat{x}_i^1(\theta_i = 1, X_0 = \underline{w}\beta/2) = \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right)\beta/2 + (\alpha/R)^{1/(1-\alpha)} \ \forall i \in [0, 0.5],$$
$$\hat{x}_i^1(\theta_i = 1, X_0 = \underline{w}(1-\beta)/2) = \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right)(1-\beta)/2 + (\alpha/R)^{1/(1-\alpha)} \ \forall i \in [0, 0.5],$$
$$\hat{x}_i^1(\theta_i = 0, X_0 = \underline{w}\beta/2) = \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right)\beta/2 \ \forall i \in [0, 0.5],$$
$$\hat{x}_i^1(\theta_i = 0, X_0 = \underline{w}(1-\beta)/2) = \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right)(1-\beta)/2 \ \forall i \in [0, 0.5],$$

and invest optimally otherwise (i.e., off the equilibrium path).

Aggregate investment in equilibrium is therefore

$$\begin{aligned} X &= X_0 + \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right) s_2 s_1/2 + (\alpha/R)^{1/(1-\alpha)} s_1/2 + \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right) s_2(1-s_1)/2 \\ &= \underline{w}s_2/2 + \left((\alpha/R)^{1/(1-\alpha)} - \underline{w}\right) s_2/2 + (\alpha/R)^{1/(1-\alpha)} s_1/2 \\ &= (\alpha/R)^{1/(1-\alpha)}(s_1 + s_2)/2, \end{aligned}$$

which is the efficient aggregate investment. Given the efficient outcome and the implied return of R on investment, all investors are indifferent between individually investing earlier or later, or investing in the safe asset, since no unilateral deviation will change X_0 due to the zero mass of all i. Hence, the described investment strategy profiles constitute a Bayesian Nash equilibrium of the dynamic investment game. □

The described efficient equilibrium works as follows: At $t = 0$, only the poor interested consumers invest in the project. Thus, at $t = 1$, the share of interested consumers in the poor consumer group is revealed via aggregate investment X_0, because the more consumers are interested in the novel good the higher the observable aggregate investment X_0. At $t = 1$ all wealthy consumers invest whatever the poor consumers wanted to but could not invest due to wealth constraints. Additionally, wealthy consumers who are interested in the project invest more to cover production of their own future demand.